Analytical prediction of the pinching mechanism of RC elements under cyclic shear using a rotation-angle softened truss model

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Abstract

The response of a reinforced concrete (RC) element under cyclic shear is characterized by the hysteretic loops of the shear stress–strain curves. These hysteretic loops can exhibit strength deterioration, stiffness degradation, and a pinched shape. Recent tests [Mansour MY, Hsu TTC. Behavior of reinforced concrete elements under cyclic shear. I: Experiments. ASCE Journal of Structural Engineering 2005;131(1):44–53] have shown that the orientation of steel grids in RC shear elements has a strong effect on the “pinching effect” in the post-yield hysteretic loops. When the steel grid was set at a 45 degree angle to the shear plane, there was no pinching effect and no strength deterioration. However, when the steel grid was set parallel to the shear plane, there was a severe pinching effect and severe strength deterioration with increasing shear strain magnitude. It was thus obvious that the undesirable “pinching effect” and strength deterioration that were attributed to the presence of high shear forces can be eliminated by properly orienting the steel grid in RC elements subjected to cyclic shear.

In this paper, two RC elements subjected to reversed cyclic shear stresses are considered to study the effect of the steel grid orientation on the shape of the cyclic shear stress–strain curves. The presence and absence of the pinching mechanism in the post-yield shear hysteretic loops is studied using the Rotating Angle Softened Truss Model (RA-STM) theory [Hsu TTC. Softened truss model theory for shear and torsion. ACI Structural Journal 1988;85(6):624–35]. It is found that the RA-STM when combined with newly proposed cyclic material constitutive relationships can rationally predict the presence and absence of the pinching effect in the shear hysteretic loops of RC shear elements but is still incapable of predicting the descending envelopes.

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1. Introduction

Structures located in earthquake regions are designed to withstand moderate seismic loading within the elastic range, and to absorb the energy of severe seismic loading using the plastic range. Consequently, it becomes necessary to evaluate the inelastic responses and energy dissipation capacities of such structures and to determine methods to enhance their seismic behavior under earthquake loading. In the case of structures that deform primarily in the flexural mode, the response is governed by well-rounded hysteretic load–deformation curves because the response of such elements is governed mainly by the properties of the reinforcing steel bars. By comparison, reinforced concrete elements that deform primarily in the shear mode frequently show significant pinching around zero load, and severe strength deterioration in their hysteretic loops.

Thus, when the shear force governs the response of a reinforced concrete (RC) element, as in the case of a low-rise shear wall, the effect of shear on the element's response is thought to be responsible for the “pinching effect” in the hysteretic loops, resulting in the degradation of stiffness, deterioration of strength and the reduction of...
energy dissipation capacity, as the cyclic loading increases beyond the yielding level. However, it was recently shown that this undesirable pinching effect [1] can be eliminated in

the hysteretic load–deformation curves of a shear-dominant element if the steel grid orientation is properly aligned in the direction of the applied principal stresses. In this case, RC shear-dominant elements can be designed to possess high energy dissipation capacities, just like RC flexural-dominant elements.

The effect of the steel bar orientation on the structural response of RC structures was first experimentally investigated by Paulay [3] and Paulay and Binney [4] who showed that the pinching effect in the hysteretic loops of coupling beams can be controlled by adding inclined shear reinforcement, as shown in Fig. 1(a). Fig. 1(b), on the other hand, shows the severe pinched shape in the hysteretic loops of the same beam considered in Fig. 1(a), when no inclined shear reinforcement is used. Their test results also revealed that the ductility, energy dissipation and strength of coupling beams were considerably improved by arranging steel diagonally.

The mechanisms behind the presence of the pinching effect in the hysteretic loops of shear-dominant structures were studied by several researchers, like Kinugasa and Nomura [5] and Fenwick et al. [6] to name a few, who showed that the pinching effect was mainly due to the opening and closing of concrete cracks under cyclic loading. According to their findings, when subjected to cyclic shear, two out of phase orthogonal sets of diagonal concrete struts and cracks form in an RC element. After steel yielding, the shear deformation significantly increases and hence the crack width also increases. When the applied shear force is reversed in direction, the opened cracks are closed in one direction and the closed cracks start to open in the orthogonal direction at very low level of shear forces. As a consequence, the strength and the stiffness of the cracked concrete are remarkably reduced during this period of load reversal, creating the pinching effect. The presence of the pinching effect was also attributed to the deterioration of the bond between concrete and the steel bars. Several bond–slip models [7–9] were in turn proposed to analytically model the pinching effect in the hysteretic loops of the load–deformation curves of shear-dominant elements.

Even though previous researchers [5, 6] were able to describe physically the pinching mechanism in shear dominant structures, to date no study has been able to show analytically the variation of the concrete and steel stresses and strains as the pinching effect progresses for the hysteretic loops of the shear stress–strain curves of RC elements. In this paper, the Rotating Angle Softened Truss Model [2] (RA-STM) is used to explain rationally the presence and absence of the pinching mechanism in the hysteretic loops of the shear stress–strain curves of two out of twelve RC elements previously tested by Mansour and Hsu [1]. The tested panels were subjected to reversed cyclic shear. The RA-STM which was previously proposed by Hsu [2] to predict the shear behavior of RC elements subjected to monotonic loading is extended in this paper to predict the cyclic behavior of two RC elements.
elements subjected to cyclic shear. This extension uses new constitutive relationships for concrete and steel under cyclic loading. Comparing the predicted responses of these two elements reveals the mechanism inherent in the pinching phenomenon.

2. Rotating angle softened truss model (RA-STM) theory

2.1. Basic principles

The RA-STM [2] was mainly developed for the analysis of reinforced concrete membrane elements subjected to in-plane monotonic stresses. The RA-STM assumes that the steel reinforcement is uniformly distributed in the concrete element and that the cracking is smeared over the element’s surfaces (Fig. 2(a)). In this investigation the theory is extended and used to predict the behavior of RC elements subjected to cyclic shear stresses. The extension of the RA-STM requires only the formulation of new material constitutive relationships for reversed cyclic loading. These constitutive relationships will be given in the next sections. The equilibrium and compatibility relationships used in this paper were originally derived by Hsu [2].

A reinforced concrete cracked element, shown in Fig. 2(a), is subjected to in-plane biaxial cyclic stresses. The directions of the longitudinal and transverse steel bars are designated by the $\ell$- and $t$- axes, respectively, constituting the $\ell$–$t$ coordinate system. Accordingly, the applied cyclic normal stresses are designated as $\sigma_\ell$ and $\sigma_t$, and the applied cyclic shear stresses as $\tau_{\ell t}$. The reinforced concrete element can be visualized as a combination of two elements with different material properties: a concrete element, shown in Fig. 2(b), and a steel element, shown in Fig. 2(c).

The principal stresses in the concrete element (without steel) can be assumed to coincide with the applied principal stresses of the reinforced concrete element before cracking. Upon cracking, the concrete element (Fig. 2(d)) is separated by the cracks into a series of concrete struts. The directions of the post-cracking principal stresses in concrete are defined by the $d$ and $r$ axes, respectively. The angle between the direction of the principal stress in the cracked concrete ($d$-axis) and the direction of the longitudinal steel ($\ell$-axis) is defined as the angle $\alpha$.

The RA-STM is based on the assumption that the inclination of principal compression stress in the concrete coincides with the inclination of the principal compression strain. The RA-STM also assumes that the cracks are smeared throughout the reinforced concrete elements, and the reinforcing bars are uniformly distributed in two
orthogonal directions (\( \ell \) and \( t \)). The behavior of the element is, therefore, formulated in terms of smeared (or average) stresses and smeared (or average) strains, so that the concept of continuum mechanics can still be applied.

When using the RA-STM to predict the structural response of a reinforced concrete element, three types of equations are needed: the equilibrium equations, the compatibility equations, and equations representing the constitutive material models. These equations are summarized in the following subsections. The derivations of all the equilibrium and compatibility equations are based on the angle \( \alpha \) (refer to Fig. 2(d)).

### 2.2. Equilibrium equations

The stresses, \( \sigma_\ell \), \( \sigma_t \), and \( \tau_\ell \), shown in Fig. 2(a), are the applied stresses on the reinforced concrete element viewed as a whole. The stresses on the concrete element alone are denoted as \( \sigma_\ell^c \), \( \sigma_t^c \) and \( \tau_\ell^c \), as shown in Fig. 2(b). The longitudinal and transverse steel provides the smeared stresses of \( \rho_\ell f_\ell \) and \( \rho_t f_t \), as shown in Fig. 2(c), where the pairs of \( \rho_\ell \) and \( \rho_t \) are the steel ratios and smeared stresses of steel bars in the \( \ell \) and \( t \) directions, respectively. The steel reinforcements are assumed to resist only axial stresses, neglecting any possible dowel action. Accordingly, the stresses in a reinforced concrete element are defined by the following three equilibrium equations:

\[
\begin{align*}
\sigma_\ell &= \sigma_\ell^c + \rho_\ell f_\ell \\
\sigma_t &= \sigma_t^c + \rho_t f_t \\
\tau_\ell &= \tau_\ell^c.
\end{align*}
\]

(1)\( \quad \)\( \quad \)\( \quad \)\( \quad \)\( \quad \)\( \quad \)

In these three equations, the three concrete stresses \( (\sigma_\ell^c, \sigma_t^c \) and \( \tau_\ell^c) \) in the \( \ell-t \) coordinate system can be related to the principal stresses of concrete \( (\sigma_d \) and \( \sigma_r) \) in the \( d-r \) coordinate system using the principle of stress transformation. The three equations can then be expressed as [2]:

\[
\begin{align*}
\sigma_\ell &= \sigma_d \cos^2 \alpha + \sigma_r \sin^2 \alpha + \rho_\ell f_\ell \\
\sigma_t &= \sigma_d \sin^2 \alpha + \sigma_r \cos^2 \alpha + \rho_t f_t \\
\tau_\ell &= (-\sigma_d + \sigma_r) \sin \alpha \cos \alpha.
\end{align*}
\]

(4)\( \quad \)\( \quad \)\( \quad \)\( \quad \)\( \quad \)\( \quad \)

(5)\( \quad \)\( \quad \)\( \quad \)\( \quad \)\( \quad \)\( \quad \)

(6)\( \quad \)\( \quad \)\( \quad \)\( \quad \)\( \quad \)\( \quad \)

### 2.3. Compatibility equations

The three compatibility equations, which represent the relationship between the strains \( (\varepsilon_\ell, \varepsilon_t \) and \( \gamma_\ell \) in the \( \ell-t \) coordinates of the reinforcement and the principal strains \( (\varepsilon_d, \varepsilon_r) \) in the \( d-r \) coordinates of the concrete principal stresses, can be shown to be [2]:

\[
\begin{align*}
\varepsilon_\ell &= \varepsilon_d \cos^2 \alpha + \varepsilon_r \sin^2 \alpha \\
\varepsilon_t &= \varepsilon_d \sin^2 \alpha + \varepsilon_r \cos^2 \alpha \\
\gamma_\ell &= (-\varepsilon_d + \varepsilon_r) \sin \alpha \cos \alpha
\end{align*}
\]

(7)\( \quad \)\( \quad \)\( \quad \)\( \quad \)\( \quad \)\( \quad \)

(8)\( \quad \)\( \quad \)\( \quad \)\( \quad \)\( \quad \)\( \quad \)

(9)\( \quad \)\( \quad \)\( \quad \)\( \quad \)\( \quad \)\( \quad \)

where \( \varepsilon_\ell \) and \( \varepsilon_t \) are the smeared principal strains of the concrete in the \( r \) and \( t \) directions, respectively.

### 2.4. Cyclic constitutive relationships of materials

The solution of Eqs. (4) through (9) requires the cyclic constitutive relationships of the constituent materials (concrete and steel) to be established. The cyclic stress–strain curves of cracked concrete in compression and tension as well as the cyclic stress–strain curves of steel bars embedded in concrete were obtained from testing full-scale RC elements as described in detail by Mansour et al. [10]. In what follows only a summary of the cyclic stress–strain curves of the materials is given.

#### 2.4.1. Cyclic constitutive relationships of cracked concrete

The cyclic stress–strain curves of cracked concrete were found to consist of several loading, unloading and reloading stages to describe the cyclic behavior of cracked concrete. The cyclic stress–strain curves of cracked concrete are shown in Fig. 3. Experimental data [10] revealed that the envelope curves for the cyclic stress–strain curves of cracked concrete in compression and tension can be approximated by the monotonic tensile and compressive stress–strain curves of concrete as proposed by Belarbi and Hsu [11]. These envelope curves are given below:

\[
\begin{align*}
\sigma_c &= \left( \xi_\sigma f'_c \right) \left( 2 - \frac{\varepsilon_c}{\xi_\sigma} \right) - \left( \frac{\varepsilon_c}{\xi_\sigma} \right)^2 \\
&+ f'_{\text{ct}} \quad \varepsilon_\sigma \leq \varepsilon_c < 0 \quad \text{(Stage C1)} \tag{10a}
\end{align*}
\]

\[
\begin{align*}
\sigma_c &= \xi_\sigma f'_c \left( 1 - \frac{\varepsilon_c}{\xi_\sigma} \frac{1}{4(\xi_\sigma - 1)} \right) \quad \varepsilon_c < \varepsilon_\sigma \quad \text{(Stage C2)} \tag{10b}
\end{align*}
\]

\[
\begin{align*}
\sigma_c &= E_c \varepsilon_c \quad 0 \leq \varepsilon_c \leq 0.00008 \quad \text{(Stage T1)} \tag{11a}
\end{align*}
\]

\[
\begin{align*}
\sigma_c &= f'_{\text{cr}} \left( \frac{0.00008}{\varepsilon_c} \right)^0.4 \quad \varepsilon_c > 0.00008 \quad \text{(Stage T2)} \tag{11b}
\end{align*}
\]

where \( \sigma_c \) is the average stress of concrete; \( \varepsilon_c \) is the average strain of concrete; \( E_c \) is the modulus of elasticity of concrete.

![Fig. 3. Stress–strain curve of concrete subjected to reversed cyclic loading.](image-url)

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Concrete taken as 3900√f′c (MPa); fcc is the cracking stress of concrete taken as 0.31√f′c (MPa); f′c and εp are the compressive strength of concrete cylinder and the peak cylinder compressive strain, respectively; f′c,t4 is the stress at point TD (in Fig. 3) between Stage C1 and Stage T4; ξσ and ξε are the softening coefficient of stress and strain, respectively, given by:

\[ ξσ = ξε = \frac{5.9}{\sqrt{f′c} (MPa)} \sqrt{1 + kεy} \]  

where εy is the principal tensile strain in the normal direction to the principal compressive direction; k is the loading coefficient taken as k = 400 for proportional loading and k = 200 for sequential loading.

For the unloading and reloading curves a linear function, given by Eq. (13) below, was proposed and found to best fit the test results [10]:

\[ σc = σci + E_{cc}(εci - εc) \]  

where σci and εci are the concrete stress and strain at the load reversal point or at the point where the stages change; E_{cc} is the modulus of elasticity of concrete which was taken to be:

\[ E_{cc} = \frac{σc - σci+1}{εc - εci+1} \]  

where σci+1 and εci+1 are the concrete stress and strain at the end of the stage under consideration.

2.4.2. Cyclic constitutive relationships of steel bars embedded in concrete

Five different stages were used to describe the cyclic stress–strain curves of steel bars embedded in concrete: Stage 1—before yielding of steel; Stage 2T—after yielding of steel in tension; Stage 2C—after yielding of steel in compression; Stage 3—unloading region; and Stage 4—reloading region. All five stages are shown illustrated in Fig. 4. Test results [10] revealed that the monotonic tensile stress–strain curves of steel bars embedded in concrete as proposed by Belarbi and Hsu [12] could also be approximated as envelope curves to the cyclic tensile stress–strain curves of steel bars. The average stress–strain relationships for Stage 1, Stage 2T and Stage 2C representing the envelope curves are given below and can be found in detail elsewhere [10]:

\[ f_s = E_sε_s \quad (ε_s ≤ ε_n) \]  

(15a)

Stage 2T

\[ f_s = f_y \left[ (0.91 - 2B) + \left( 0.02 + 0.25B \frac{ε_i}{ε_y} \right) \right] (ε_s > ε_n) \]  

(15b)

Stage 2C

\[ f_s = -f_y \quad (f_s ≤ -f_y) \]  

(15c)

where f_s and ε_s are the average stress and strain of mild steel bars, respectively; f_y and ε_y are the yield stress and strain of bare mild steel bars, respectively; E_s is the modulus of elasticity of steel bars; ρ is the reinforcement steel ratio; ε_n = ε_y(0.93 - 2B); and B = (f_cr/f_y)^1.5/ρ.

The proposed cyclic stress–strain relationships for an embedded steel bar in concrete include the unloading and reloading branches taking into account the Baushinger effect. Several models [13,14] have been developed for the cyclic stress–strain relationship of steel bars. In this paper, the unloading and reloading branches of the cyclic stress–strain relationship are given by the following equation [15]:

\[ \epsilon_s - \epsilon_i = \frac{f_s - f_i}{E_s} \left[ 1 + A^R \left( \frac{f_s - f_i}{f_y} \right)^{R-1} \right] \]  

(15d)

where f_i and ε_i are the average steel stress and strain at the load reversal point, respectively. The coefficients A and R in Eq. (15d) were determined from reversed cyclic loading tests to best fit the test results of Mansour et al. [10] and were taken as A = 1.9k_p^{-1.1}, R = 10k_p^{-0.2}. The parameter k_p is the plastic strain ε_p normalized by ε_n and is taken as k_p = ε_p/ε_n.

2.5. Solution procedure

The solution procedure used in the analysis is given in the flow chart shown in Fig. 5. The non-linear solution was controlled by means of a step-wise increment in the value of the principal strain ε_d according to a prescribed deformation history. An iterative procedure was performed for each value of ε_d until all the equilibrium, compatibility and constitutive equations were satisfied.

3. Tests of elements CA3 and CE3

The two test elements considered, CA3 and CE3, are shown in Fig. 6(a). Two coordinate systems (H, V) and
angle of 45° with the $H$–$V$ coordinate system, while in element CE3 both the $H$–$V$ and $\ell$–$t$ coordinate systems coincide with each other. Both panels had equal amount of steel in the longitudinal ($\ell$) and transverse ($t$) directions. The reinforcing ratios of elements CA3 and CE3 are 1.70% and 1.20%, respectively, in each direction. The material properties for these two elements are summarized in Table 1.

The loads were applied through 20 horizontal and 20 vertical in-plane jacks, and the horizontal and vertical applied stresses were measured from the 40 jack load cells. The in-plane pure shear stress in the 45° direction of the element was then calculated by stress transformation. The average (or smeared) strains were measured continuously using a set of LVDTs mounted on the two faces of the test element as shown in Fig. 6(b). Each LVDT measured a displacement over a length that traversed at least several cracks. The measurement of the average strains in the horizontal, vertical and one diagonal direction (at 45° to the horizontal direction) can allow for the calculation of the shear strain at 45° ($\gamma_{45°}$) using the principle of strain transformation as shown below:

$$
\gamma_{45°} = \varepsilon_H - \varepsilon_V
$$

where $\varepsilon_H$ and $\varepsilon_V$ are the measured average strains in the horizontal and vertical directions, respectively.

The elements were subjected to reversed cyclic principal stresses in the horizontal and vertical directions. When these two principal applied stresses were equal in magnitude and opposite in direction, a state of pure shear stress $\tau_{45°}$ was created at the 45° direction to the applied principal stresses. The average (or smeared) stresses and strains were continuously obtained by measuring the jack forces and the element deformations during testing. Under cyclic loading, the shear stresses alternated between positive and negative values with increasing magnitude, as shown in Fig. 7. The absolute values of maximum shear stresses (positive and negative) were chosen to be multiples of the cracking shear stress. Once the yielding load of the RC element was approached, a strain-control mode was adopted until failure of the element. During this strain-control mode, the shear strain was used as an input signal to control the principal horizontal stress which in turn was used to control the principal vertical stress. Both stresses were always maintained equal in magnitude and opposite in direction during this stage. The shear strains were chosen to be multiples of the yield shear strain, as shown in Fig. 7. More details on the testing procedures and the test panels can be found elsewhere [1].

4. Experimental and predicted cyclic shear stress–strain curves

The experimental shear hysteretic loops of the two elements, CA3 (45°) and CE3 (0°), are shown in Fig. 8. In this figure, the vertical and horizontal axes represent the shear stress $\tau_{45°}$ and the shear strain $\gamma_{45°}$ at 45° to
Table 1
Specification of specimens and material properties

<table>
<thead>
<tr>
<th>Panel</th>
<th>Concrete</th>
<th>Steel in $t$ direction</th>
<th>Steel in $\ell$ direction</th>
<th>Steel angle$^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Rebars ($\rho_\ell$)</td>
<td>$f_y$ ($f'_{\ell}$)</td>
<td>Rebars ($\rho_t$)</td>
</tr>
<tr>
<td>CA3</td>
<td>45</td>
<td>No. 6 (284)</td>
<td>1.70</td>
<td>428</td>
</tr>
<tr>
<td>CE3</td>
<td>50</td>
<td>No. 6 (284)</td>
<td>1.20</td>
<td>428</td>
</tr>
</tbody>
</table>

$^a$ The angle is measured with respect to the horizontal direction.

Fig. 7. Loading history of CA3 ($45^\circ$).

Fig. 8. Comparison between the observed and predicted cyclic shear stress–strain curves of panels CA3 ($45^\circ$) and CE3 ($0^\circ$): (a) test result of CA3, (b) analyzed result of CA3, (c) test result of CE3, and (d) analyzed result of CE3.
the principal coordinate of applied stresses. In Fig. 8, the hysteretic loops of element CA3 displayed a highly pinched shape that is generally associated with shear dominated behavior. The envelope curve of this element also exhibited a distinct descending branch indicating a severe strength degradation of the element with increasing shear strain magnitude. In contrast, no pinching effect can be observed in the hysteretic loops of element CE3, with its steel grid parallel to the applied principal stresses. The envelope curve of element CE3 did not have a descending branch and the strength deterioration was not noticeable.

Fig. 8 also compares the experimental cyclic shear stress–strain relationships of CA3 and CE3 to those predicted using the RA-STM. The predicted results of the RA-STM are in good agreement with the observed cyclic shear stress–strain curves of CA3 and CE3.

The RA-STM can thus be used successfully to predict the presence and absence of the pinching effect and the pre-peak behavior accurately but remains incapable of predicting the descending envelopes as in the case of element CA3.

5. Behavior of materials

To study the effect of the steel grid orientation on the behavior of the materials for the two considered elements, the predicted stress–strain curves of the steel bars in the longitudinal direction and that of concrete in one of the principal directions are considered.

5.1. Predicted cyclic stress–strain curves of steel bars embedded in concrete

The cyclic stress–strain curves of the steel bars in the longitudinal direction of elements CA3 and CE3 are shown in Fig. 9. These cyclic hysteretic loops were predicted using the solution procedure of the RA-STM previously outlined and the material constitutive relationships of steel bars embedded in concrete as given by Eqs. (15a) through (15d).

Fig. 9 shows that the cyclic stress–strain curves of the longitudinal steel bars in element CA3 are always tensile and are never compressive, meaning that the applied compression stresses needed to produce a state of pure shear had to be solely resisted by concrete. On the other hand, the hysteretic loops of the stress–strain curves of the steel in element CE3 show that the longitudinal steel bars are subjected to both compression and tension stresses, as the intensity of cyclic load increases. Therefore, when the steel grid orientation in a shear element is set parallel to the plane of pure shear, as is the case of element CA3, the steel bars have to be designed to resist not only tensile stresses but also compressive stresses. However, for shear elements with steel grids set parallel to the plane of pure shear, as is the case of element CE3, the steel bars have to be designed to resist tensile stresses only while the concrete will resist the compressive stresses.

5.2. Predicted cyclic stress–strain curves of concrete

The predicted cyclic stress–strain curves of cracked concrete in one of the principal direction for elements CA3 and CE3 are shown in Fig. 10. The figures show a reduction in the maximum attained compressive strengths of concrete for elements CA3 and CE3, calculated from Eq. (12), when compared to their corresponding concrete cylinder strengths. For example, the maximum compressive strength of the concrete of element CA3 is almost 15 MPa, which is about 33% of its maximum concrete strength cylinder. This reduction in concrete compressive strength is attributed mainly to the presence of cracks in the orthogonal direction. Fig. 10 also shows that the cyclic stress–strain curves of concrete of element CE3 were mainly in tension, meaning that the cracks in this element did not fully close since the compressive stresses in the concrete were relatively small and insufficient to cause the cracks to fully close.

6. Presence and absence of the pinching mechanism

To better understand the presence and absence of the pinching mechanism in the hysteretic loops of the shear stress–strain curves, the predicted results of the two elements considered in this paper with two different steel grid orientations are analyzed using the RA-STM. Element CA3
Fig. 10. Predicted axial cyclic stress–strain curves of concrete of panels CA3 \( (45^\circ) \) and CE3 \( (0^\circ) \) in the longitudinal direction using the RA-STM.

Fig. 11. One cycle of hysteresis for CA3 \( (45^\circ) \): (a) shear stress vs. strain curve, (b) stress vs. strain curve of steel in the \( \ell \) or \( t \) directions, (c) stress vs. strain curve of concrete in the \( H \) direction, and (d) stress vs. strain curve of concrete in the \( V \) direction.

\( (45^\circ) \) is chosen to explain the presence of the pinching mechanism, while element CE3 \( (0^\circ) \) is chosen to explain the absence of the pinching mechanism.

6.1. Pinching mechanism in element CA3 \( (45^\circ) \)

The predicted results of element CA3 are considered to study the presence of the pinching mechanisms. Fig. 11(a) shows the first cycle of the hysteretic loop after yielding in terms of the shear stress \( \tau_{45^\circ} \) vs. the shear strain \( \gamma_{45^\circ} \) in the \( 45^\circ \) direction. Fig. 11(b) shows the corresponding stress–strain curves of the steel bars in the \( \ell \) and \( t \) directions, while Fig. 11(c) and (d) show the corresponding stress–strain curves of the concrete in the horizontal and vertical directions, respectively.

To correlate the shear stresses and shear strains to the stresses and strains in the steel bars, in the horizontal concrete and in the vertical concrete, four points A, B, C, and D are chosen in Fig. 11(a) and the corresponding points are designated in Fig. 11(b), (c) and (d). Point A is at the maximum shear strain of the first cycle beyond yielding. Point B is at the stage where the shear stress is zero after unloading. Point C is taken in the negative strain region at the end of the low-stress pinching zone just before the sudden increase in stiffness. Point D is at the maximum negative shear strain of the first negative cycle beyond yielding. The calculated stresses and strains at points A, B, C and D are shown in Table 2.

As the element is unloaded from point A to point B in Fig. 11(a), the steel stress also is reduced from the
maximum tensile stress to almost zero stress, Fig. 11(b). Correspondingly, the concrete horizontal strain is unloaded from a maximum positive value of 0.01081 to a smaller positive value of 0.00627, Fig. 11(c). In this unloading stage, the vertical cracks due to the horizontal strain are closing from a very large width to a much smaller width under a negligible horizontal concrete stress. At the same time, the concrete in the vertical direction is unloading from a maximum compressive (negative) stress value to almost zero, Fig. 11(d). Note that at point B, the shear stress, the steel stress, and the concrete vertical and horizontal stresses are all close to zero. During this unloading stage, all the equilibrium as well as compatibility equations are satisfied.

As the shear stress–strain curve proceeds from point B to point C, Fig. 11(a), the horizontal strain decreases from 0.00627 to nearly zero, Fig. 11(c), representing the closing of the vertical cracks due to a reducing horizontal strain. At the same time, the vertical strain increases from almost zero to about 0.00625, Fig. 11(d), representing an opening of the horizontal cracks due to vertical strain. From compatibility, these two strains (horizontal and vertical) produce a large change in the shear strains from +0.00648 to −0.00610, but only a slight increase of strain in the steel bars, Fig. 11(b). From equilibrium, the small stresses in the steel bars and in the concrete (horizontal and the vertical) produce a small shear stress. In other words, the element is offering very small shear resistance to a very large shear strain. This phenomenon characterizes the pinching effect.

As the shear stress–strain curve proceeds from point C to point D, the horizontal strain becomes compression and the cracks in the vertical direction are completely closed. The horizontal compressive stress then increases rapidly following the reloading constitutive relationship, Fig. 11(c). At the same time, the vertical strain increases rapidly, Fig. 11(d), causing a rapid opening of the horizontal cracks. From compatibility, the large vertical strain produces a large increase of strain in the steel bars, Fig. 11(b). From equilibrium, the large stresses in the steel bars and in the vertical concrete produce a large shear stress. In other words, the element is again offering a large shear stiffness. The three segments of curves from point A to point D in Fig. 11(a) clearly define the pinched shape of the hysteretic loops.

### 6.2. Absence of pinching mechanism in element CE3 (0°)

The predicted results of element CE3 are analyzed to illustrate the absence of a pinching mechanism when the steel bars are oriented in the direction of the principal stresses. Fig. 12(a) shows the first cycle of the hysteretic loop after yielding in terms of shear stress vs. shear strain. Fig. 12(b) and (c) show the stress–strain curves of steel bars in the horizontal and vertical directions, respectively. Fig. 12(d) and (e) show the stress–strain curves of concrete in the horizontal and vertical directions, respectively.

To correlate the shear stresses and shear strains to the stresses and strains in the steel bars and in the concrete, four points A, B, C, and D are chosen in Fig. 12(a) and designated on Fig. 12(b), (c), (d) and (e). Point A is at the maximum shear strain of the second cycle beyond yielding. Point B is at the stage where the shear stress is zero after unloading. Point C is taken in the negative strain region when the steel reaches yielding. Point D is at the maximum negative shear strain of the second negative cycle after steel yielding. The calculated stresses and strains at points A, B, C and D are shown in Table 3.

As the element is unloaded from point A to point B in Fig. 12(a), the shear stress is reduced from 5.31 to 0.05 MPa (almost zero). The longitudinal steel stress also is reduced from the maximum tensile stress (442 MPa) to almost zero stress (−1.63 MPa), as shown in Fig. 12(b). Correspondingly, the horizontal concrete stress is unloaded from a maximum positive value of 0.24 MPa to a smaller positive value of 0.10 MPa, while the concrete horizontal strain is unloaded from a maximum positive value of 0.1654 to a smaller positive value of 0.0042, as shown in Fig. 12(d). In this unloading stage (region A to B), the vertical cracks due to the horizontal strain are closing from a very large width to a much smaller width under a negligible horizontal concrete stress. At the same time, the concrete in the vertical direction is unloaded from a compressive stress value of −0.70 MPa to almost zero (0.05 MPa), as

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<th>Calculated values</th>
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Table 2
Predicted results of panel CA3 (steel angle is 45°)

As the shear stress–strain curve proceeds from point C to point D, the horizontal strain becomes compression and the vertical strain increases from almost zero to about 0.00625, Fig. 11(d), representing an opening of the horizontal cracks due to vertical strain. From compatibility, these two strains (horizontal and vertical) produce a large change in the shear strains from +0.00648 to −0.00610, but only a slight increase of strain in the steel bars, Fig. 11(b). From equilibrium, the small stresses in the steel bars and in the concrete (horizontal and the vertical) produce a small shear stress. In other words, the element is offering very small shear resistance to a very large shear strain. This phenomenon characterizes the pinching effect.

As the shear stress–strain curve proceeds from point C to point D, the horizontal strain becomes compression and the cracks in the vertical direction are completely closed. The horizontal compressive stress then increases rapidly following the reloading constitutive relationship, Fig. 11(c). At the same time, the vertical strain increases rapidly, Fig. 11(d), causing a rapid opening of the horizontal cracks. From compatibility, the large vertical strain produces a large increase of strain in the steel bars, Fig. 11(b). From equilibrium, the large stresses in the steel bars and in the vertical concrete produce a large shear stress. In other words, the element is again offering a large shear stiffness. The three
shown in Fig. 12(e). Note that, at point B, the shear stress, the longitudinal and transverse steel stresses, as well as the vertical and horizontal concrete stresses, are all close to zero.

As the shear stress–strain curve proceeds from point B to point C, Fig. 12(a), the horizontal strain decreases from 0.00142 to 0.000675 (nearly zero), Fig. 12(d). In this stage the vertical cracks are still closing and are associated with very low concrete horizontal stresses (Fig. 12(d)). However, this change in horizontal strain from 0.00142 to 0.000675 causes high compressive stresses (−376.8 MPa) to build up in the longitudinal (or horizontal) reinforcing bars (Fig. 11(b)). At the same time, the vertical strains increase from 0.00555 (almost zero) to about 0.01182, as shown in Fig. 12(e), representing an opening of the horizontal cracks. This opening of the horizontal cracks occurs at very low vertical concrete stresses (Fig. 12(e)) but causes high tensile stresses (392.9 MPa) to build up in the transverse (or vertical) reinforcing bars (Fig. 12(c)). From compatibility, these strains (horizontal and vertical) produce a large change in the shear strains from +0.00867 to −0.00507, and from equilibrium, the large steel stresses in the longitudinal (horizontal) and transverse (vertical) steel bars produce a large jump in the shear stress from almost a zero value at point B to a large negative value of −4.67 MPa at point C (Fig. 12(a)). In other words, the element is offering a high shear stiffness from point B to point C (Fig. 12(a)). This phenomenon characterizes the absence of the pinching mechanism.

6.3. Physical visualization of pinching mechanism

The presence of the pinching mechanism in element CA3 (45°), as shown by region BC in Fig. 11, can be explained intuitively by examining a cracked element with 45° steel bars, as shown in Fig. 13(a). A state of pure shear stress in the 45° direction of this element in region BC is equivalent to applying a horizontal compressive stress \(\sigma_H\) and a vertical tensile stress \(\sigma_V\) of equal magnitude. In the reverse loading stage from point B to point C (Fig. 11(a)), which defines the region where pinching occurs, both the vertical and the
Table 3
Predicted results of panel CE3 (steel angle is 0°)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Equations</th>
<th>Calculated values</th>
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a Transformation equations.

Fig. 13. Cracked RC element with 45° steel grid orientation: (a) RC panel, (b) compressive stress in the horizontal direction, and (c) tensile stress in the vertical direction.

horizontal cracks are open. The concrete struts have not yet been formed, and the applied stresses \(\sigma_H\) and \(\sigma_V\) must be resisted by the two 45° steel bars since the carrying capacity of cracked concrete can be neglected. Separate the effect of \(\sigma_H\) and \(\sigma_V\), as shown in Fig. 13(b) and (c), respectively. The horizontal stress \(\sigma_H\) will induce a compressive stress in the two 45° steel bars, Fig. 13(b), while the vertical tensile stress \(\sigma_V\) will induce a tensile stress of equal magnitude in the same two 45° bars, Fig. 13(c). These two stresses in the two 45° steel bars will cancel out. As a result, the element offers no shear resistance to the applied shear stress \(\tau_{45^\circ}\) in the 45° direction, while the shear strain \(\gamma_{45^\circ}\) increases rapidly due to the opening of the horizontal cracks and the closing of the vertical cracks. The resulting near zero shear stiffness in the BC regions creates the pinching effect in the hysteretic loops of the shear stress vs. shear strain curves.

6.4. Physical visualization of absence of pinching mechanism

The absence of the pinching mechanism in element CE3 (90°) can be intuitively visualized by considering a cracked element with 90° steel bars, as shown in Fig. 14(a). A state of pure shear stress in the 45° direction can be achieved by applying a horizontal compressive stress \(\sigma_H\) and vertical tensile stress \(\sigma_V\) of equal magnitude. In the reverse loading stage from point B to point C (Fig. 12(a)), both the vertical and horizontal cracks are open. Consequently, the horizontal compressive stress \(\sigma_H\) is resisted by the compressive stress in the horizontal steel bar only (Fig. 14(b)), while the vertical tensile stress \(\sigma_V\) is resisted by a tensile stress in the vertical bar only (Fig. 14(c)). The shear stress \(\tau_{45^\circ}\) in the 45° direction is then contributed by the compressive stress in the horizontal steel bars and the tensile stress in the vertical steel bars. As a result, the shear stress increases proportionally to the shear strain, and the shear stiffness in the BC region will be large, thus creating a smooth and robust hysteretic loop without the pinched shape.

7. Conclusions

In this paper, two RC membrane elements (panels) subjected to cyclic shear were considered to assess the effect of the steel grid orientation on the “pinching effect” for the hysteretic loops of the shear stress–strain curves.

Experimentally, the test results of the two panels showed that orienting the steel grid in the direction of the principal applied stresses eliminates the undesirable pinching effect and increases the energy dissipation capacity and ductility of RC elements subjected to cyclic shear.

Theoretically, the RA-STM theory was shown to predict the cyclic shear responses of the two considered panels with
good accuracy once combined with accurate material cyclic constitutive relationships. The RA-STM was found capable of predicting the pinched shape in the hysteretic loops of the shear stress–strain curves, as well as the stresses and strains at yield, and the unloading and reloading shear stiffness of the two panels. The RA-STM was also used to explain the presence or the absence of a pinched shape in the shear hysteretic loops by showing the variation of the steel and concrete stresses during one loading cycle. However, it is worth mentioning that there is still an area of weakness in the RA-STM: the model cannot yet be used to predict the descending envelope curves. To take care of this weakness, the model has to take into account Poisson’s effect as well as the deterioration of concrete under cyclic loading [16].

Finally, since the pinching effect is influenced by the orientation of the steel grid in RC shear-dominant elements, this undesirable pinching mechanism can be eliminated if the steel reinforcement is placed parallel to the direction of the applied principal stresses. This will provide a means to enhance the ductility of shear elements, so they will have higher energy dissipation capacity to resist earthquake loading.

References


